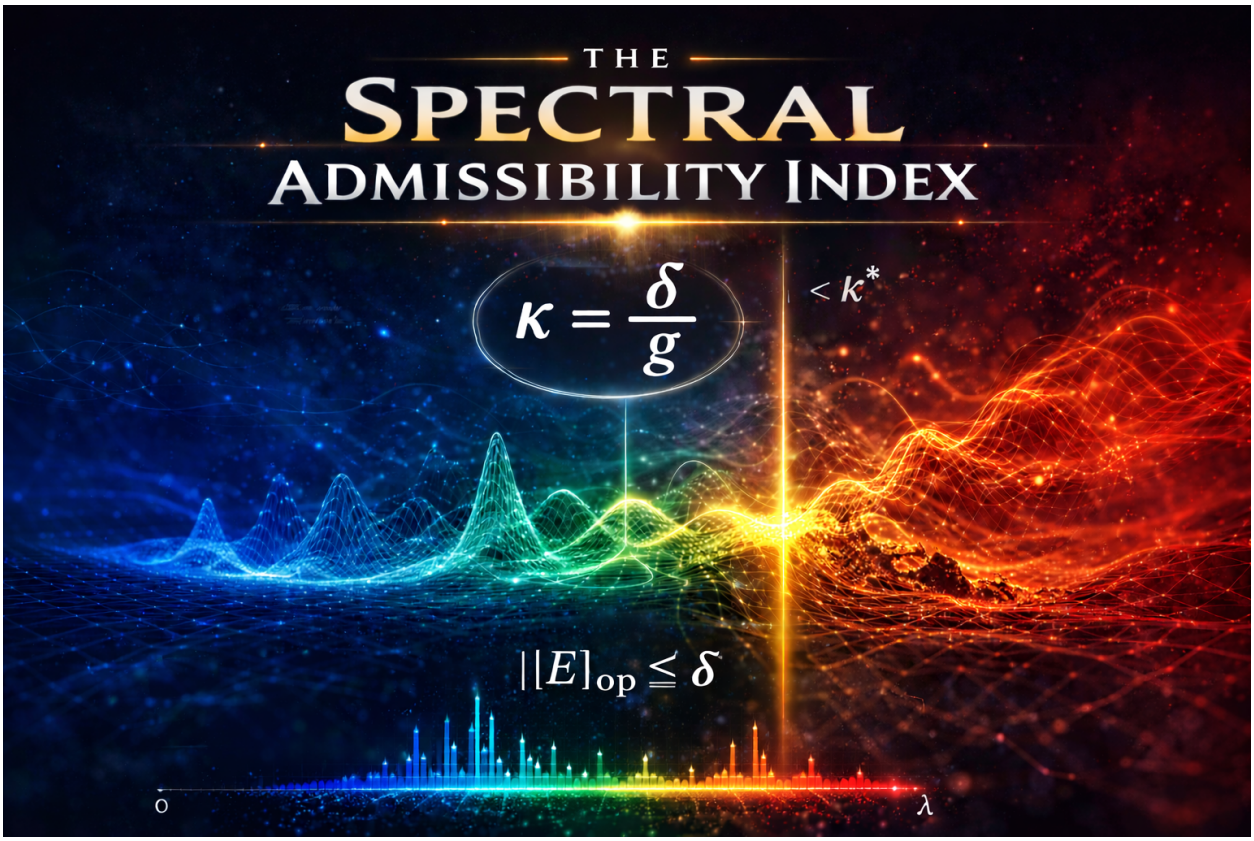
THE
SPECTRAL
ADMISSIBILITY INDEX
$\kappa = \frac{\delta}{g}$
$< \kappa^*$
$\|[E]_{op} \leq \delta$
0
λ

WHITEWOLF PAPER #19

THE SPECTRAL ADMISSIBILITY INDEX

A Harmonic Regime Analysis of Constrained Transport

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Whitewolf Papers — Song 2 Tone 6

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ABSTRACT

This paper introduces the Spectral Admissibility Index, a dimensionless invariant governing when low-band spectral classification remains structurally meaningful under constrained transport modeled by nonnegative self-adjoint operators.

Given spectral gap g and perturbation magnitude \delta, we define

\kappa := \frac{\delta}{g}.

For a chosen notion of spectral identity (e.g., projector distance or principal angle), classification remains invariant only in regimes satisfying \kappa < \kappa_*, where \kappa_* depends explicitly on the stability criterion adopted. Beyond this boundary, low-band identity becomes perturbation-sensitive and regime naming loses invariant meaning.

While gap-relative perturbation bounds are classical, this paper abstracts the ratio \kappa into a portable diagnostic that governs when harmonic regime language is structurally admissible across stability notions. The result is not a new physical theory, but a formal identification of the boundary separating invariant classification from regime reconfiguration within an operator lens.

1. ORIENTATION

Whitewolf Paper #18 established geometric transport invariants independent of dynamical interpretation.

The present paper introduces a spectral operator lens and identifies the structural condition under which harmonic regime classification remains invariant.

This work:

- Does not modify geometric transport results
- Does not replace gravitational theory
- Does not introduce operational mechanisms

It isolates a classification admissibility boundary within spectral transport analysis.

2. OPERATOR SETTING AND SCOPE

Let:

- (X,m) be a σ-finite measure space
- $\mathcal{H} = L^2(X,m)$
- $L : \mathrm{Dom}(L) \subset \mathcal{H} \to \mathcal{H}$

with L self-adjoint and nonnegative.

This assumption is natural for Laplacian-type and symmetric diffusion generators. Non-self-adjoint transport generators (e.g., drift-diffusion operators) require pseudospectral or singular-value analysis and are outside the present scope. The admissibility index introduced here applies only in the self-adjoint setting.

Assume a discrete spectral decomposition for clarity:

$$L \phi_k = \lambda_k \phi_k, \quad 0 = \lambda_0 \le \lambda_1 \le \dots$$

Define the low-band projector:

$$P_K := \mathbf{1}_{[0,\lambda_K]}(L).$$

“Identity” in this paper means the invariant low-band subspace

\operatorname{Ran}(P_K),

measured via operator-norm distance or principal angle between spectral projectors.

3. LOW-BAND CONCENTRATION

For $f \in \mathcal{H}$, define

$$D_K(f) = \frac{\|P_K f\|^2}{\|f\|^2}.$$

Under dissipative evolution

$$\partial_t u = -L u,$$

higher modes decay monotonically and

$$u(t) \to P_0 u(0)$$

as $t \to \infty$.

Thus low-band concentration may emerge under relaxation dynamics.

In practice, however, the generator L is known only approximately. Any modeling, numerical, or physical perturbation E introduces ambiguity in spectral identity. The admissibility index introduced below quantifies when low-band concentration remains invariant under such perturbations.

4. GAP PROTECTION

Assume a spectral gap:

$$g := \lambda_{K+1} - \lambda_K > 0.$$

Let perturbations satisfy:

$$L' = L + E, \quad \|E\|_{\mathrm{op}} \le \delta.$$

Classical results (e.g., Davis–Kahan sinΘ theorem [Davis–Kahan 1970]) provide bounds on the distance between spectral projectors under gap-relative perturbation. In projector-norm formulation:

$$\|P'_K - P_K\|_{\mathrm{op}} \le C \frac{\delta}{g},$$

for universal constant C.

Rather than fixing constants specific to one formulation, we abstract the gap-relative ratio itself.

5. THE SPECTRAL ADMISSIBILITY INDEX

Definition (Spectral Admissibility Index)

Given spectral gap g and perturbation magnitude \delta, define

\kappa := \frac{\delta}{g}.

Definition (Spectral Admissibility Boundary)

Let \kappa_* denote the threshold associated with a chosen stability notion (e.g., projector norm, principal angle, eigenvalue labeling).

Then:

- Admissible regime: $\kappa < \kappa^*$

  Low-band identity remains invariant under the selected stability criterion.

- Swap regime: $\kappa \geq \kappa^*$

Subspace rotation or eigenvalue crossing may occur; regime identity becomes perturbation-sensitive.

In the canonical projector-norm Davis–Kahan setting, one may take $\kappa_* \sim 1/2$ up to universal constants. Other stability notions yield different constants. The index κ makes this dependence explicit rather than implicit.

Crossing the boundary $\kappa = \kappa_*$ permits subspace rotation and mode reclassification. The transition marks the point at which spectral naming ceases to be stability-protected.

The novelty of this paper is not the stability lemma itself, but the abstraction of gap-relative perturbation into a portable invariant governing when spectral regime classification is admissible across stability notions.

6. HARMONIC REGIME LANGUAGE

A harmonic regime, in the context of this paper, refers to a situation in which low-band concentration both emerges and remains invariant under admissible perturbations.

The term “harmonic” is used in the operator-theoretic sense of spectral decomposition. It does not introduce additional physical content beyond the operator framework defined above.

The Spectral Admissibility Index provides the structural condition under which such regime language is justified.

For $\kappa < \kappa_*$, dominance statements refer to a stability-protected subspace and retain invariant meaning.

For $\kappa \ge \kappa_*$, the same language becomes perturbation-dependent. At that point, classification ceases to be structurally neutral and must be reconsidered relative to the perturbation model.

The index therefore does not describe a phenomenon. It governs when description itself remains admissible.

7. FAILURE MODES

The framework fails if:

1. No meaningful self-adjoint generator describes transport.
2. No spectral gap protects the low band.
3. Perturbations routinely yield \kappa \ge \kappa_*.
4. Observables inject irreducible high-frequency mass.
5. Concentration does not persist under relaxation.

8. RELATION TO PRIOR WORK

Whitewolf Paper #18 established geometric transport invariants.

The present paper introduces a spectral meta-layer: it determines when harmonic classification of transport remains structurally admissible.

The Davis–Kahan theorem and related stability results are classical. The contribution here is the identification of the dimensionless index

$$\kappa = \frac{\delta}{g}$$

as a portable diagnostic separating invariant classification from regime reconfiguration across stability notions.

9. CONCLUSION

Transport geometry establishes propagation structure.

Spectral analysis reveals concentration tendencies.

For $\kappa < \kappa_*$, harmonic regime classification is stability-protected.

Beyond that boundary, regime identity becomes perturbation-dependent.

The Spectral Admissibility Index formalizes this boundary.

No further claims are made.

REFERENCES

Davis, C., and Kahan, W. M.

"The rotation of eigenvectors by a perturbation. III."

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10. A Human Walk-Through of Spectral Admissibility

Earlier in this paper, we defined a boundary condition for classification.

We introduced:

- A self-adjoint operator L
- Its eigenvalues λ_k
- The spectral gap $g = \lambda_{K+1} - \lambda_K$
- A perturbation magnitude δ
- And the Spectral Admissibility Index
  $\kappa = \frac{\delta}{g}$.

The claim was simple:

When separation dominates disturbance, modal identity remains stable.

When disturbance approaches separation, identity becomes ambiguous.

The following demonstration makes that statement visible.

You have likely seen this before.

A thin metal plate mounted above a speaker.

Fine sand spread across the surface.

A tone plays beneath it.

At certain frequencies, the sand gathers into sharp geometric patterns.

At others, it does not.

These are Chladni patterns — visible traces of vibration modes.

What You Are Seeing (In Paper Language)

Take a square steel plate:

- ~25 cm side length
- 1–2 mm thick
- Lightly clamped
- Sand evenly distributed
- Speaker beneath

Sweep a sine wave from ~50 Hz upward.

At discrete resonances, clean nodal lines appear.

In the language of this paper:

The plate is exhibiting eigenfunctions ϕ_k of an operator L.

The sand geometry is the spatial trace of ϕ_k.

The resonant frequencies correspond to eigenvalues λ_k.

The sand is simply revealing what the spectrum already contains.

Where the Spectral Gap Appears

Not all resonances are equally spaced.

Some neighboring modes are well separated.

Some lie close together.

That spacing is the physical analogue of the spectral gap:

$g = \lambda_{K+1} - \lambda_K$.

When the gap is large, one nodal pattern dominates clearly.

When the gap is small, modal identity becomes sensitive.

This is exactly the separation parameter defined earlier.

Where the Perturbation Appears

Increase amplitude slightly.

Shift the clamp.

Change the contact point of the driver.

These introduce disturbance.

In the formal framework of the paper, this is the perturbation magnitude δ.

Now observe:

If disturbance remains small relative to separation, the sand pattern remains stable.

If disturbance becomes comparable to separation, patterns reorganize abruptly.

That transition corresponds precisely to

$$\kappa = \frac{\delta}{g}.$$

When $\kappa \ll 1$, modal classification is stable.

When $\kappa \approx 1$, modal identity becomes ambiguous.

What This Bridge Does

The sand demonstration does not derive the index.

It does not prove the theorem.

It shows what the theorem describes:

Spectral separation protects identity.

Disturbance relative to separation destabilizes it.

The operator framework formalizes this boundary.

The sand makes it visible.

APPENDIX:

INTERACTIVE DEMONSTRATION LAYER

(Spectral Regime Rendering — Operator-Faithful)

B.0 Scope and Boundary

This appendix does not extend the theory.

It does not reinterpret transport physically.

It does not claim equivalence between operator spectra and sound or image.

It does one thing only:

It allows the reader to interact directly with structural analogues of the quantities defined in the main body:

- Spectral gap g
- Perturbation magnitude δ
- The ratio $\kappa = \delta / g$

Earlier in this paper, we formalized when harmonic regime language is admissible.

Here, you will see and hear what that boundary feels like.

Nothing more.

B.1 Platform One — Direct Spectral Instantiation

(Pure Frequency Structure)

Platform:

https://hzgenerator.com/

This tool allows multiple simultaneous tones with independent control of:

- Frequency
- Waveform (sine, triangle, square, saw)
- Amplitude
- Stereo panning

There is no compositional engine.

No stylistic inference.

Only raw spectral structure.

B.1.1 Stable Regime — \kappa < \kappa_*

From Section B.1 of the main appendix:

L = \mathrm{diag}(0,4,9,16)

Low band K = 1

g = 9 - 4 = 5

Let \delta = 1

\kappa = 1/5 = 0.2

Admissible regime.

Instantiate the Spectrum

Add three tones:

- 220 Hz (sine, ~33%, center)
- 330 Hz (sine, ~33%, center)
- 440 Hz (sine, ~33%, center)

These arise from:

$110 \times \sqrt{\lambda_k}$

You are not constructing music.

You are instantiating square-root spectral lifts.

What to Notice

Because $\kappa \ll 1$:

- Intervals feel stable.
- No strong beating collapse.

- Harmonic center remains coherent.
- No modal competition.

This is spectral identity protected by separation.

Here:

- Frequency spacing represents g.
- Minor detuning represents \delta.
- Coherence stability represents \kappa.

B.1.2 Approaching the Boundary — \kappa \approx \kappa_*

Simulate increasing perturbation:

- Shift 330 Hz slightly (±4–8 Hz)
- Shift 440 Hz slightly (±4–8 Hz)
- Introduce slight amplitude imbalance (±5–10%)

Now observe:

- Beating emerges.
- Dominance clarity weakens.
- Harmonic center destabilizes.

You are increasing \delta relative to g.

B.1.3 Crossing the Boundary — \kappa > \kappa_*

Increase detuning significantly:

- 330 Hz → drift within 310–350 Hz
- 440 Hz → drift within 400–480 Hz
- Change one waveform to triangle or saw
- Increase amplitude asymmetry

Now:

- No stable low-band anchor.
- Coherence collapses.
- Structure reorganizes perceptually.

This is regime reconfiguration.

The sound is not dramatic.

It is diagnostic.

B.2 Platform Two — Modal Field Visualization

(Cymatic Structural Fields)

Platform:

https://cymavis.com/

This platform renders frequency as:

- Chladni plate nodal structures
- 3D water surface displacement
- Radial and lattice modal fields

Unlike the first platform (which renders spectral superposition), this one renders modal geometry directly.

B.2.1 Stable Regime

Select:

- Chladni Plate mode
- Fixed frequency (e.g., 220 Hz or 440 Hz)

Observe:

- Clean nodal lines
- Symmetric geometry
- Stable modal identity

In operator language:

- Each pattern corresponds to an eigenfunction \phi_k.
- Spatial symmetry reflects spectral isolation.
- Large effective separation mimics large g.

B.2.2 Increasing Perturbation

Now:

- Slightly shift frequency
- Increase volume
- Switch between pattern modes
- Use 3D Water mode and vary frequency gradually

Observe:

- Pattern instability
- Nodal line migration
- Symmetry breakdown
- Rapid reorganization

This corresponds to:

- Reduced effective gap
- Increased perturbation sensitivity
- Approaching \kappa_*

When small frequency shifts cause large geometric reconfiguration, the system is near boundary.

Structural Interpretation

Earlier we defined:

$\kappa = \delta / g$

In this visualization:

- g corresponds to modal spacing.
- δ corresponds to frequency perturbation.
- Pattern stability corresponds to admissibility.

The sand plate in Section 10 showed this physically.

Cymavis shows it computationally.

B.3 Platform Three — Spectral Decomposition & Reconstruction

(Fourier Structural Synthesis)

Platform:

https://stilesjesus.github.io/fourier_visualizer/

This platform displays:

- Component sine waves
- Their amplitudes
- Their superposition
- Real-time reconstruction

Unlike Platform One (direct instantiation), this one shows:

How structure emerges from components.

This mirrors the operator framework more directly than either prior platform.

B.3.1 Stable Regime

Construct a signal using:

- Three strong low-frequency components
- Well-separated frequencies
- Minimal amplitude imbalance

Observe:

- Clean waveform reconstruction
- Predictable composite structure
- Stable periodicity

Here:

- Separation between dominant components represents g.
- Small coefficient noise represents \delta.
- Clean reconstruction reflects admissibility.

B.3.2 Near Boundary

Now:

- Add nearby frequency components
- Increase amplitude noise
- Introduce closely spaced frequencies

Observe:

- Beating
- Envelope modulation
- Loss of clean periodic dominance

You are reducing effective separation.

You are increasing \delta.

B.3.3 Crossing Boundary

Add:

- Many closely spaced components
- Random amplitude weights
- Minor phase offsets

Now:

- Structure becomes irregular.
- No dominant low band.
- Composite loses stable identity.

This is spectral regime collapse.

B.4 Cross-Platform Structural Mapping

Paper Quantity	hzgenerator	Cymavis	Fourier Visualizer
Spectral gap g	Frequency spacing	Modal separation	Component spacing
Perturbation \delta	Detuning	Frequency shift / instability	Amplitude / phase noise
\kappa	Audible coherence stability	Pattern stability	Reconstruction dominance
Boundary crossing	Harmonic collapse	Rapid nodal reorganization	Loss of periodic identity

Each platform shows the same structural principle through a different interface:

1. Superposition stability
2. Modal geometry
3. Spectral reconstruction

None assert ontology.

All demonstrate admissibility.

B.5 Why Three Platforms

The main body formalizes:

When classification is structurally admissible.

Section 10 made it visible physically.

Appendix B makes it interactive:

- Platform One: hear separation.
- Platform Two: see modal geometry.
- Platform Three: see synthesis and collapse.

The invariant survives format change.

That is the point.

B.6 Closing

These demonstrations do not extend theory.

They do not imply physics.

They do not prove anything new.

They allow the reader to experience:

- Separation dominance.
- Perturbation scaling.
- Stability boundary crossing.

Earlier we wrote:

These tools let you feel that boundary directly.

End - Interactive Demonstration Layer

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